Problem 1.26

The hallmark of an inertial reference frame is that any object which is subject to zero net force will travel in a straight line at constant speed. To illustrate this, consider the following: I am standing on a level floor at the origin of an inertial frame $\mathcal{S}$ and kick a frictionless puck due north across the floor. **(a)** Write down the x and y coordinates of the puck as functions of time as seen from my inertial frame. (Use x and y axes pointing east and north respectively.) Now consider two more observers, the first at rest in a frame $\mathcal{S}'$ that travels with constant velocity v due east relative to $\mathcal{S}$, the second at rest in a frame $\mathcal{S}''$ that travels with constant *acceleration* due east relative to $\mathcal{S}$. (All three frames coincide at the moment when I kick the puck, and $\mathcal{S}''$ is at rest relative to $\mathcal{S}$ at that same moment.) **(b)** Find the coordinates x', y' of the puck and describe the puck's path as seen from $\mathcal{S}'$. **(c)** Do the same for $\mathcal{S}''$. Which of the frames is inertial?

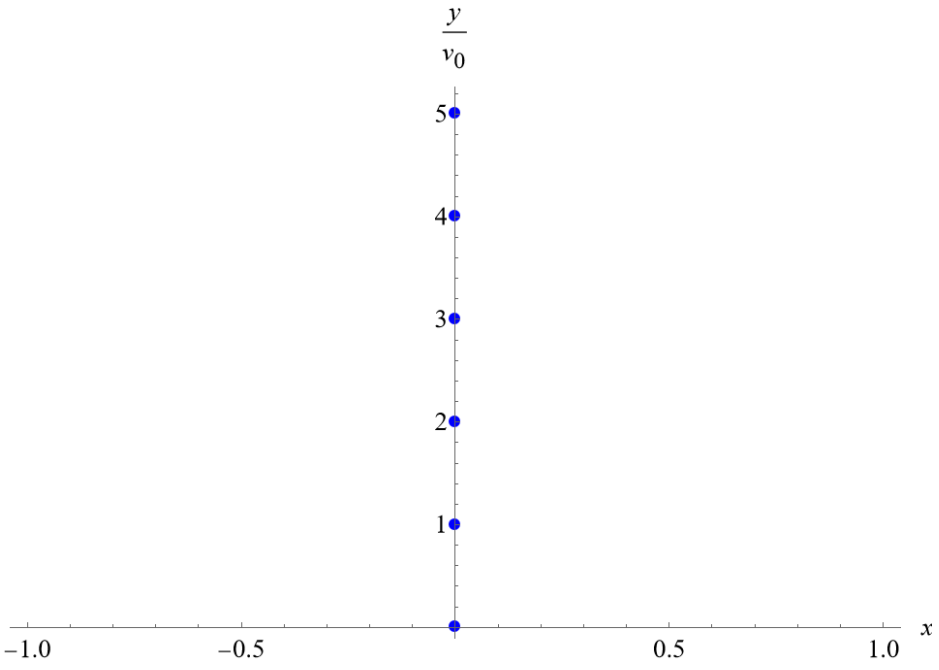
Solution

Part (a)

The observer is standing at the origin in the frame $\mathcal{S}$, which stays still.

$$\begin{cases} v_x = 0 \quad \rightarrow \quad \dfrac{dx}{dt} = 0 \quad \rightarrow \quad x = x_0 \quad \rightarrow \quad x = 0 \\ v_y = v_0 \quad \rightarrow \quad \dfrac{dy}{dt} = v_0 \quad \rightarrow \quad y = v_0 t + y_0 \quad \rightarrow \quad y = v_0 t \end{cases}$$

Below is a motion diagram for the frictionless puck from the frame $\mathcal{S}$. Snapshots are taken every second.



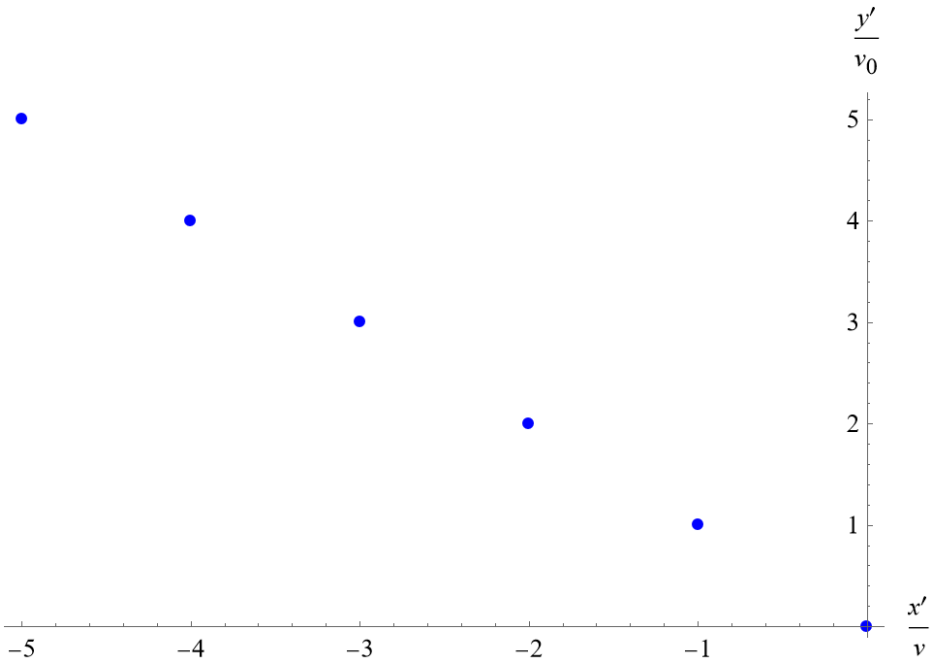
$\mathcal{S}$ is an inertial frame because it neither accelerates nor rotates.

Part (b)

The observer is standing at the origin in the frame $\mathcal{S}'$, which moves to the east with constant speed v. From the observer's point of view, the puck moves to the west with speed v in addition to moving to the north at speed v_0.

$$\begin{cases} v'_x = -v \quad \rightarrow \quad \dfrac{dx'}{dt} = -v \quad \rightarrow \quad x' = -vt + x_0 \quad \rightarrow \quad x' = -vt \\ v'_y = v_0 \quad \rightarrow \quad \dfrac{dy'}{dt} = v_0 \quad \rightarrow \quad y' = v_0 t + y_0 \quad \rightarrow \quad y' = v_0 t \end{cases}$$

Below is a motion diagram for the frictionless puck from the frame $\mathcal{S}'$. Snapshots are taken every second.



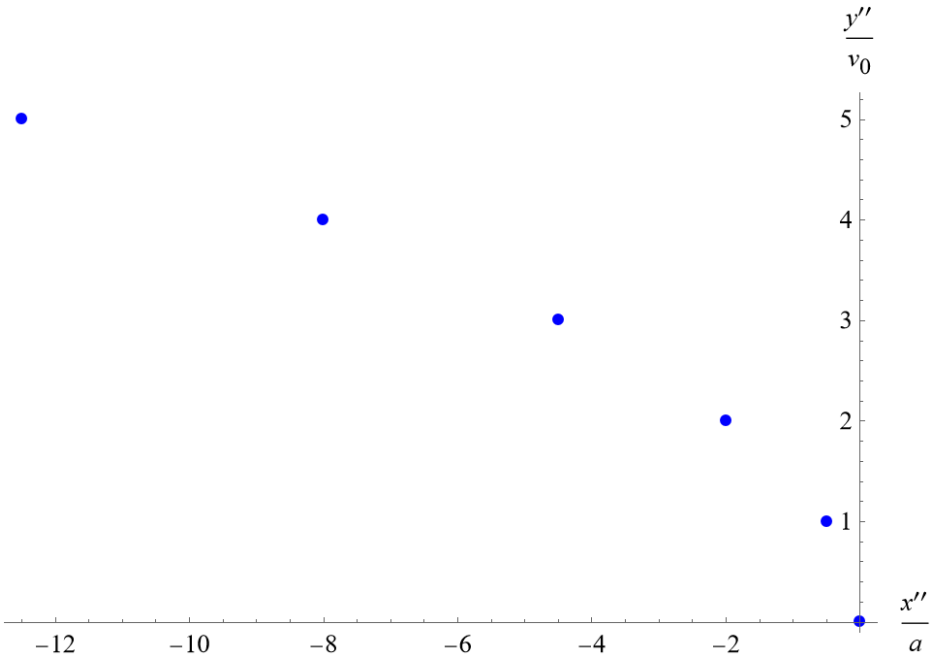
$\mathcal{S}'$ is an inertial frame because it neither accelerates nor rotates.

Part (c)

The observer is standing at the origin in the frame $\mathcal{S}''$, which moves to the east with constant acceleration a. From the observer's point of view, the puck accelerates to the west with acceleration a in addition to moving to the north at speed v_0.

$$\begin{cases} a_x'' = -a \quad \rightarrow \quad \dfrac{d^2x''}{dt^2} = -a \quad \rightarrow \quad x'' = -\dfrac{1}{2}at^2 + x_0 \quad \rightarrow \quad x'' = -\dfrac{1}{2}at^2 \\ v_y'' = v_0 \quad \rightarrow \quad \dfrac{dy''}{dt} = v_0 \quad \rightarrow \quad y'' = v_0 t + y_0 \quad \rightarrow \quad y'' = v_0 t \end{cases}$$

Below is a motion diagram for the frictionless puck from the frame $\mathcal{S}''$. Snapshots are taken every second.



$\mathcal{S}''$ is not an inertial frame because it accelerates.